ACHARYA A. V. PATEL JR. COLLEGE

EXCELLENCE PROGRAM-SYJC (COMMERCE), 2019-20

SYNOPSIS

MATHEMATICS AND STATISTICS - PART 1

DIFFERENTIATION **[08 MARKS FOR H.S.C.]**

DEFINITION: If y = f(x) for all real values of x, then $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$, if the limit exists, is called the derivative of f w.r.t. x, at x and is denoted by f '(x) or $\frac{dy}{dx}$.

RULES OF DIFFERENTIATION:-

If u and v are differentiable functions of x then

I. $y = u \pm v$, then $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

II. $y = u.v$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

III. $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

IV. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ **(chain rule)**

DERIVATIVES OF SOME STANDARD FUNCTIONS:-

- $\frac{d}{dx}(k) = 0$, k is constant
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x.\cot x$
- $\frac{d}{dx}(\log x) = \frac{1}{x}$
- $\frac{d}{dx}(a^x) = a^x \log a$ $\qquad \frac{d}{dx}(e^x) = e^x$

DERIVATIVES OF COMPOSITE FUNCTIONS;-

- $\frac{d}{dx}(f(x))^n = n[f(x)]^{n-1}$
- $\frac{d}{dx}\sin(f(x)) = \cos(f(x)) \ \frac{d}{dx}f(x)$
- $\frac{d}{dx}\cos(f(x)) = -\sin(f(x)) \ \frac{d}{dx}f(x)$
- $\frac{d}{dx}(\tan f(x)) = \sec^2 f(x)\frac{d}{dx}f(x)$
- $\frac{d}{dx}(\cot f(x)) = -\cos ec^2 f(x)\frac{d}{dx}f(x)$
- $\frac{d}{dx}(\cot f(x)) = -\cos ec^2 f(x)\frac{d}{dx}f(x)$
- $\frac{d}{dx}\sec f(x) = \sec f(x)\tan f(x)\frac{d}{dx}f(x)$
- $\frac{d}{dx}(\cos ecf(x)) = -\cos ecf(x).\cot f(x) \ \frac{d}{dx}f(x)$
- $\frac{d}{dx}(\log f(x)) = \frac{1}{f(x)}\frac{d}{dx}f(x)$
- $\frac{d}{dx}(a^{f(x)}) = a^{f(x)}\log a \ \ \frac{d}{dx}f(x)$
- $\frac{d}{dx}(e^{f(x)}) = e^{f(x)}\frac{d}{dx}f(x)$

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DERIVATIVE OF INVERSE FUNCTIONS:

1) $\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}, -1<x<1$

2) $\frac{d}{dx}(\cos^{-1}x)=-\frac{1}{\sqrt{1-x^2}}, \quad -1<x<1$

3) $\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}, \quad x\varepsilon R.$

4) $\frac{d}{dx}(\cot^{-1}x)=-\frac{1}{1+x^2}$, for $x\varepsilon R$

5) $\frac{d(\sec^{-1}x)}{dx}=\frac{1}{+\left|x\sqrt{x^2-1}\right|}$ where $|x|>1$.

6) $\frac{d}{dx}(\operatorname{cosec}^{-1}x)=\frac{-1}{\left|x\sqrt{x^2-1}\right|}$, where $|x|>1$.

Specific substitution for solving the expression:

Expression	Substitution
$\sqrt{1-x^2}$	$x=\sin\theta$ or $x=\cos\theta$
$\sqrt{1+x^2}$	$x=\tan\theta$ or $x=\cot\theta$
$\sqrt{x^2-1}$	$x=\sec\theta$ or $x=\operatorname{cosec}\theta$
$1-2x^2$	$x=\sin\theta$
$2x^2-1$	$x=\cos\theta$
$\frac{2x}{1+x^2}$	$x=\tan\theta$
$\frac{1-x^2}{1+x^2}$	$x=\tan\theta$
$\frac{2x}{1-x^2}$	$x=\tan\theta$

LOGARITHMIC DIFFERENTIATION:

To differentiate the **function of the type $[f(x)]^{g(x)}$**

Take the log on the both side of the equation and then differentiate w.r.t 'x'.

i.e. If $y = [f(x)]^{g(x)}$

$\therefore \log y = g(x) \log f(x)$

Apply following method in case of product or quotient of two or more functions.

i.e. 1. $y = f_1(x).f_2(x)$

$\therefore \log y = \log f_1(x) + \log f_2(x)$

2. $y = \dfrac{f_1(x).f_2(x)}{g_1(x).g_2(x)}$

$\therefore \log y = \log f_1(x) + \log f_2(x) - \log g_1(x) - \log g_2(x)$

After this differentiate w.r.t 'x'

Formulae:

- $\dfrac{d}{dx}(\log x) = \dfrac{1}{x}$
- $\dfrac{d}{dx}(\log f(x)) = \dfrac{1}{f(x)} \dfrac{d}{dx} f(x)$

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IMPLICIT FUNCTION DIFFERENTIATION:

There are expressions of the type $f(x,y) = c, c = constant$,such expressions are implicit functions.

Example: $ax^2 + 2hxy + by^2 = 0$ or $x^2y + xy^2 + \sin(xy) = 0$ or any other expressions of this type.

1) **Differentiate the equation w.r.t x.**
2) **Find equation for $\frac{dy}{dx}$.**

DERIVATIVE OF PARAMETRIC FUNCTIONS:

If $x = f(\theta)$ and $y = g(\theta)$ are differentiable functions of θ then $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$.

Steps:

1) **Find derivative of $x = f(\theta)$, w.r.t θ**
2) **Find derivative of $y = g(\theta)$, w.r.t θ**
3) **Replace $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ in $\frac{dy}{dx}$**

Where θ is common parameter in both functions, $x = f(\theta)$ and $y = g(\theta)$.

SECOND ORDER DERIVATIVES:

$If\ y = f(x)$

1) $\frac{dy}{dx} = f'(x)$ first order derivative
2) $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$ second order derivative

Steps:

1) **Find $\frac{dy}{dx}$**
2) **Again find $\frac{d}{dx}$ of above answer.**